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91. Proposed by **F. P. MATZ**, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

There are two unequal square numbers the sum of whose sum, difference, product, and quotient, is a square. Find the two numbers.

Solution by **J. H. DRUMMOND**, LL. D., Portland, Me.

Any two unequal squares answer the conditions of the question; for, let a^2 and b^2 be the numbers, then $a^2+b^2+a^2-b^2+a^2b^2+(a^2/b^2)$ must be a square. Reducing, and dropping the factor a^2, we have $2+b^2+(1/b^2)$, a square. This is readily put under the form $\frac{b^4+2b^2+1}{b^2}$, which is the square of $\frac{b^2+1}{b^2}$.

Q. E. D.

Also solved in a similar manner by *G. B. M. ZERR*, *J. SCHEFFER*, and *H. S. VANDIVER*.

92. Proposed by **L. C. WALKER**, A.M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

(*a*) Find the least three integral numbers such that the difference of every two of them shall be a square number; (*b*) Find the least three square numbers such that the difference of every two of them shall be a square number.

Solution by **J. H. DRUMMOND**, LL. D., Portland, Me.

Part 2. Let x^2, m^2x^2, and n^2x^2 be the three numbers; then m^2-1, n^2-1, and m^2-n^2 must all be squares.

Assume $m=\frac{p^2+1}{p^2-1}$ and $n=\frac{q^2+1}{q^2-1}$, and m^2-1 and n^2-1 will be squares and it remains to make $\left[\frac{p^2+1}{p^2-1}\right]^2-\left[\frac{q^2+1}{q^2-1}\right]=\square$, or reducing $(p^2q^2-1)(\frac{q^2}{p^2}-1)=\square$. This is done by making $pq=\frac{r^2+s^2}{2rs}$ and $\frac{q}{p}=\frac{t^2+u^2}{2tu}$.

But $pq\times q/p=q^2=\left[\frac{r^2+s^2}{2rs}\right]\left[\frac{t^2+u^2}{2tu}\right]$. Hence $rs(r^2+s^2)tu(t^2+u^2)$ must be a square. Take $r=f+g$, $s=f-g$, $t=h+k$, and $u=h-k$, and substituting and reducing, we have $(f^4-g^4)(h^4-k^4)=\square$. Assume $f^2=h^4-k^4+v^2$ and $g^2=h^4-k^4-v^2$ and $(f^4-g^4)=4v^2(h^4-k^4)$, and $(f^4-g^4)(h^4-k^4)$ becomes $4v^2(h^4-k^4)^2$, a square. Assume $h^2=a^2v^2$ and $k^4=b^2v^2$; then $h^2=av$ and $k^2=bv$. But $f^2-g^2=2v^2$. Assume $f+g=4v$ and $f-g=\frac{1}{2}v$, then $f=\frac{9}{4}v$ and $g=\frac{7}{4}v$. Then $r=4v$ and $s=\frac{1}{2}v$. Then $pq\left[=\frac{r^2+s^2}{2rs}\right]=\frac{65}{16}$, and $q/p=\frac{t^2+u^2}{2tu}$ $=\frac{2(h^2+k^2)}{2(h^2-k^2)}=\frac{a+b}{a-b}$; and $h^4-k^4+v^2=f^2=\frac{81v^2}{16}$, and $h^4-k^4=\frac{65v^2}{16}$. Therefore $a^2-b^2=\frac{h^4-k^4}{v^2}=\frac{65}{16}$. Now $pq\times q/p=q^2=\frac{65(a+b)}{16(a-b)}=\frac{65(a+b)^2}{16(a-b)^2}=(a+b)^2$ and $q=(a+b)$ and $p=(a-b)$. Hence $m=\frac{(a-b)^2+1}{(a-b)^2-1}$ and $n=\frac{(a+b)^2+1}{(a+b)^2-1}$, in which a and b may be any square numbers which make $a^2-b^2=\frac{65}{16}$.

It would seem that $a^2-b^2=\frac{65}{16}$ ought to lead readily to a general solution, but a and b were both so taken that they must be squares; b is readily found to be $\frac{13-5c^2}{8c}$. Hence $26c-10c^3$ must be a square; it is evident that this is the case when $c=1$. Then $b=1$ and $a=\frac{9}{4}$. Substituting these in the values of m and n, and we have $m=\frac{41}{9}$ and $n=\frac{185}{153}$. Taking $x=153$, we have $mx=697$, and $nx=185$, and the numbers are $(153)^2$, $(185)^2$, and $(697)^2$.

Also solved by *G. B. M. ZERR*, and *EDWARD D. GRABER*.

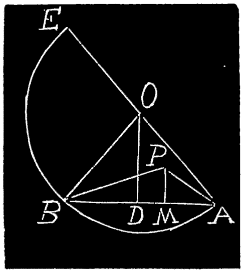
AVERAGE AND PROBABILITY.

111. Proposed by **LON C. WALKER, A.M.**, Professor of Mathematics, Petaluma High School, Petaluma, Cal.

If a radius be drawn at random in a given semi-circle, and a point taken at random in one of the sectors formed, show that the chance that a random line drawn through the point will cut the arc of the sector is $1-\frac{1}{\pi^2}\log 2$.

Solution by the PROPOSER.

Let ABE be the given semicircle, OB the random radius, P the random point, OD and PM perpendicular to AB.



Put $DM=x$, $PM=y$, $OA=1$, $\angle AOB=\theta$, $\angle APM=\phi$, $\angle BPM=\psi$. Then $AD=\sin\frac{1}{2}\theta$, $OD=\cos\frac{1}{2}\theta$, area of segment $ACB=\frac{1}{2}(\theta-\sin\theta)$, $\phi=\tan^{-1}\left(\frac{\sin\frac{1}{2}\theta-x}{y}\right)$, $\psi=\tan^{-1}\left(\frac{\sin\frac{1}{2}\theta+x}{y}\right)$

When P is in the segment ACB the random line will cut the arc whatever be its direction, and when P is in the triangle AOB the number of favorable directions of the random line will be $2(\phi+\psi)$. Hence we have

$$p=\frac{\int_0^\pi \pi(\theta-\sin\theta)d\theta+\int_0^\pi\int_0^{\cos\frac{1}{2}\theta}\int_{-\tan\frac{1}{2}\theta(\cos\frac{1}{2}\theta-y)}^{\tan\frac{1}{2}\theta(\cos\frac{1}{2}\theta-y)}2(\phi+\psi)dxdyd\theta}{\int_0^\pi \pi\theta d\theta}$$

$$=1-\frac{4}{\pi^2}+\frac{4}{\pi^3}\int_0^\pi\int_0^{\cos\frac{1}{2}\theta}\Bigg[2\tan\tfrac{1}{2}\theta(2\cos\tfrac{1}{2}\theta-y)\tan^{-1}\tan\tfrac{1}{2}\theta\left(\frac{2\cos\frac{1}{2}\theta-y}{y}\right)$$

$$-y\theta\tan\tfrac{1}{2}\theta-y\log\left(\frac{y^2+\tan^2\frac{1}{2}\theta(2\cos\frac{1}{2}\theta-y)^2}{2y^2}\right)\Bigg]dy\,d\theta$$

$$=1-\frac{4}{\pi^2}+\frac{4}{\pi^3}\int_0^\pi\Bigg[\theta\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-2(\theta-\pi)\sin^3\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-2\sin^2\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta\log^2$$

$$+\tfrac{1}{4}\log\left(\frac{1+\cos\theta}{1-\cos\theta}\right)+\tfrac{1}{4}\cos\theta\log(1+\cos\theta)+\tfrac{1}{2}\cos2\theta\log(1-\cos\theta)\Bigg]d\theta=1-\frac{1}{\pi^2}\log 2.$$

Solved with same result by *F. P. MATZ*. Professor Zerr gets as a result $1-(1/4\pi^2)(8\log 2+7)$.